Section 6.1: Composite functions

1) f(x) = 3x - 6 g(x) = 2x + 10

1a) $(f \circ g)(x)$ = 3(2x+10) – 6 = 6x + 30 – 6 Answer: $(f \circ g)(x)$ = 6x+24	1b) the domain of $(f \circ g)(x)$ The domain of both f and g are all real numbers, so there is no work to find the domain of part a. Answer: domain $(-\infty, \infty)$

1c) $(g \circ f)(x)$ = 2(3x – 6) + 10 = 6x – 12 + 10 Answer: $(g \circ f)(x)$ = 6x -2	1d) the domain of $(g \circ f)(x)$ The domain of both f and g are all real numbers, so there is no work to find the domain of part c. Answer: domain $(-\infty, \infty)$

3) f(x) = x^2 + 5 g(x) = 3x – 4

3a) $(f \circ g)(x)$ = $(3x-4)^2$ +5 = (3x – 4)(3x – 4) + 5 = $9x^2$ – 12x – 12x + 16 + 5 = $9x^2$ – 24x + 21 = 3($3x^2$ - 8x + 7) (this doesn't factor more) Answer: $(f \circ g)(x)$ = 3($3x^2$ – 8x + 7)	3b) the domain of $(f \circ g)(x)$ The domain of both f and g are all real numbers, so there is no work to find the domain of part a. Answer: domain $(-\infty, \infty)$

3c) $(g \circ f)(x)$ = 3(x^2 + 5) – 4 = $3x^2$ + 15 – 4 Answer: $(g \circ f)(x)$ = $3x^2$ + 11	3d) the domain of $(g \circ f)(x)$ The domain of both f and g are all real numbers, so there is no work to find the domain of part c. Answer: domain $(-\infty, \infty)$

5) f(x) = x – 4 g(x) = x^2 + 2x- 1

5a) $(f \circ g)(x)$	5b) the domain of $(f \circ g)(x)$
= (x^2 + 2x – 1) – 4 = x^2 + 2x – 1 – 4 Answer: $(f \circ g)(x)$ = x^2 + 2x – 5	The domain of both f and g are all real numbers, so there is no work to find the domain of part a. Answer: domain $(-\infty, \infty)$

5c) $(g \circ f)(x)$	5d) the domain of $(g \circ f)(x)$
= (x-4)^2 + 2(x-4) – 1 = x^2 – 4x – 4x + 16 + 2x – 8 – 1 = x^2 – 6x +7 (this is prime and can't be factored) Answer: $(g \circ f)(x)$ = x^2 – 6x + 7	The domain of both f and g are all real numbers, so there is no work to find the domain of part c. Answer: domain $(-\infty, \infty)$

7) $f(x) = \frac{2}{x+4}$ g(x) = $\frac{3}{x-7}$

7a) $(f \circ g)(x)$	7b) the domain of $(f \circ g)(x)$ = f(g(x))
= $\frac{2}{\frac{3}{x-7}+4}$ = $\frac{2}{\frac{3}{x-7}+\frac{4(x-7)}{x-7}}$ = $\frac{2}{\frac{3+4x-28}{x-7}}$ = $\frac{2(x-7)}{4x-25}$ = $\frac{2(x-7)}{4x-25}$ Answer: $(f \circ g)(x)$ = $\frac{2(x-7)}{4x-25}$	First note that the domain of g(x) is all real numbers except 7, so we exclude 7 from the domain of $(f \circ g)(x)$. To find the number to exclude from the domain of g(x) just set the denominator equal to zero and solve for x. x - 7 = 0 X = 7 In addition the answer to part a has a domain of all real numbers except, so 25/4 needs to be excluded from the domain as well. To find the number to exclude from the domain of $(f \circ g)(x)$ just set the denominator of the answer equal to zero and solve for x. 4x – 25 = 0 4x = 25 x = 25/4 Answer: domain of $(f \circ g)(x)$ is all real numbers except 7 and 25/4.

7c) $(g \circ f)(x)$

$= \frac{3}{\frac{2}{x+4} - 7}$

$= \frac{3}{\frac{2}{x+4} - \frac{7(x+4)}{x+4}}$

$= \frac{3}{\frac{2-7x-28}{x+4}}$

$= \frac{3(x+4)}{-7x-26}$

Answer: $(g \circ f)(x) = \frac{3(x+4)}{-7x-26}$

7d) the domain of $(g \circ f)(x)$
the domain of $(g \circ f)(x)$
First note that the domain of f(x) is all real numbers except -4, so we exclude -4 from the domain of $(g \circ f)(x)$.

To find the number to exclude from the domain of f(x) just set the denominator equal to zero and solve for x.

x + 4 = 0
x = -4

In addition the answer to part c has a domain of all real numbers except 26/7, so 26/7 needs to be excluded from the domain as well.

To find the number to exclude from the domain of $(g \circ f)(x)$ just set the denominator of the answer equal to zero and solve for x.

-7x - 26 = 0
-26 = 7x
-26/7 = x

Answer: domain of $(g \circ f)(x)$ is all real numbers except -4 and -26/7

9) $f(x) = \frac{1}{x-3}$ $g(x) = \frac{1}{x}$

9a) $(f \circ g)(x)$ = f(g(x))	9b) the domain of $(f \circ g)(x)$
= $\frac{1}{\frac{1}{x}-3}$ = $\frac{1}{\frac{1}{x}-\frac{x}{x}*3}$ = $\frac{1}{\frac{1-3x}{x}}$ Answer: $(f \circ g)(x)$ = $\frac{x}{-3x+1}$	First note that the domain of g(x) is all real numbers except 0, so we exclude 0 from the domain of $(f \circ g)(x)$. To find the number to exclude from the domain of g(x) just set the denominator equal to zero and solve for x. x = 0 In addition the answer to part a has a domain of all real numbers except 1/3, so 1/3 needs to be excluded from the domain as well. To find the number to exclude from the domain of $(f \circ g)(x)$ just set the denominator of the answer equal to zero and solve for x. -3x+ 1 = 0 1 = 3x 1/3 = x Answer: domain of $(f \circ g)(x)$ is all real numbers except 0 and 1/3

9c) $(g \circ f)(x)$	9d) the domain of $(g \circ f)(x)$
$= \frac{1}{\frac{1}{x-3}}$ $= \frac{1(x-3)}{1}$ Answer: $(g \circ f)(x)$ = x - 3	the domain of $(g \circ f)(x)$ First note that the domain of f(x) is all real numbers except 3, so we exclude 3 from the domain of $(g \circ f)(x)$. To find the number to exclude from the domain of f(x) just set the denominator equal to zero and solve for x. x - 3 = 0 x = 3 In addition the answer to part c has a domain of all real numbers, so nothing more needs to be excluded from the domain.. Answer: domain of $(g \circ f)(x)$ is all real numbers except 3

11) f(x) = 7x + 1 g(x) = $\frac{x-1}{7}$

11a) show $(f \circ g)(x)$ = x	11b) show $(g \circ f)(x)$ = x
$(f \circ g)(x)$ = $7(g(x))$ $(f \circ g)(x)$ = $7\left(\frac{x-1}{7}\right) + 1$ = x – 1 + 1 = x	$(g \circ f)(x)$ = $\frac{f(x)-1}{7}$ $(g \circ f)(x)$ = $\frac{(7x+1)-1}{7} = \frac{7x+1-1}{7} = \frac{7x}{x}$ = x

13) $f(x) = \frac{x-5}{2}$ $g(x) = 2x + 5$

13a) show $(f \circ g)(x)$ = x	13b) *show* $(g \circ f)(x)$ = x
$(f \circ g)(x)$ = $\frac{g(x)-5}{2}$ $(f \circ g)(x)$ = $\frac{2x+5-5}{2} = \frac{2x}{2}$ = x	$(g \circ f)(x)$ = $2(f(x)) + 5$ $(g \circ f)(x)$ = $2\left(\frac{x-5}{2}\right) + 5$ = x-5+5 = x